THE DOUBLE LAYER IN AN INSULATOR SUBJECT TO SHOCK COMPRESSION

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It has been shown [1] that a polar dielectric produces charges on the surfaces perpendicular to the direction of propagation of a shock wave; a current passes in the circuit of Fig. 1, in which 1 is the fixed plate with guard ring, 2 is the moving plate, and 3 is the insulator, the direction of incidence being shown by the arrows.

The initial current step is proportional to the dipole moment of the molecule, the number of dipoles per unit volume, and the plate area S, while it is inversely proportional to the thickness l_0. The current is produced by rotation of the dipoles.

The effect in, say, lucite can be used in dynamic pressure transducers. Here we assume that a double layer is formed behind the shock-wave front; the equivalent circuit is discussed.

1. EQUIVALENT CIRCUIT OF A POLARIZATION TRANSDUCER

We assume [1] that the wave polarizes the material, e.g., by rotating the dipoles, but that the state persists only while a dipole is accelerated, with subsequent relaxation and return to an equilibrium state, which has altered thermodynamic parameters. A polarized layer thus exists around the front.

A double layer moving at a constant speed in the circuit of Fig. 1 is equivalent to an emf E_0 determined by the thickness δ of the layer, the degree of orientation of the dipoles, and so on. This source, in general, possesses an internal resistance R_i, since in the steady state it cannot produce a current greater than a value governed by the number of dipoles oriented in unit time.

Figure 2 shows the x–t diagram, in which OA is the path of the plate (mass speed u), OB is the path of the shock wave (double layer, wave speed D), region 1 is uncompressed material (dielectric constant ε_1, bulk specific resistance ρ_1), and region 2 is compressed material (ε_2 and ρ_2).

We consider the case of $\delta \ll l_0$; then for any time $0 \leq t \leq l_0/D$ the system of Fig. 1 may be interpreted via the circuit of Fig. 3a. Switch T is closed at t = 0 and opens at $t = l_0/D$. The circuit parameters with subscripts 1 and 2 relate respectively to regions 1 and 2 in Fig. 2 and are written for unit area in the form

$$C_1 = \frac{\varepsilon_1}{4\pi(l_0 - Dt)}, \qquad C_2 = \frac{\varepsilon_2\sigma}{4\pi Dt}, \quad R_1 = \rho_1(l_0 - Dt),$$
$$R_2 = \frac{\rho_2 Dt}{\sigma} \qquad \left(\sigma = \frac{D}{D-u}\right) \tag{1.1}$$

in which σ is compression. The circuit of Fig. 3a is described by a second-order linear differential equation, which is difficult to solve. We consider two particular cases, in which R_1 is such that its shunt action on C_1 can be neglected.

Case 1. The conductivity behind the shock front is small. We neglect the shunting action of R_2 and pass from the equivalent circuit of Fig. 3a to that of Fig. 3b, in which

$$R = R_i + R_e, \qquad C = C_1C_2(C_1 + C_2)^{-1}. \tag{1.2}$$

For this we have

$$E_0 = iR + V_c, \quad \text{or} \quad I + \varphi = 1$$
$$(\varphi = V_cE_0^{-1},\ I = iRE_0^{-1}). \tag{1.3}$$

We differentiate the equation relating charge to voltage for a capacitor to get

$$I = \varphi R\left(\frac{C}{\varphi}\frac{d\varphi}{dt} + \frac{dC}{dt}\right). \tag{1.4}$$

Here the reciprocal of the quantity in parentheses can be considered as the effective resistance of the capacitor, which is denoted by R_c, while R_c/R is denoted by $2\mu^2$. From (1.4), $R_c = dt/dC$ if φ = constant. From Eqs. (1.1) and (1.2),

$$\mu = 2\sqrt{\pi}\,\frac{l_0 - (1-z)Dt}{\sqrt{2RD\varepsilon_1(1-z)}} \qquad \left(z = \frac{\varepsilon_1}{\varepsilon_2\sigma}\right). \tag{1.5}$$

Substitution of Eq. (1.4) into Eq. (1.3) and replacement of t by μ from Eq. (1.5) gives

$$\frac{d\varphi}{d\mu} - \left(2\mu + \frac{1}{\mu}\right)\varphi = -2\mu. \tag{1.6}$$

The general solution to this is

$$\varphi = \mu\exp(\mu^2)\,[\mathrm{const} - \sqrt{\pi}\,\mathrm{erf}(\mu)]. \tag{1.7}$$

The value of μ for t = 0 is

$$\mu_0 = 2\sqrt{\pi}\,l_0\,[2RD\varepsilon_1(1-z)]^{-1/2} \tag{1.8}$$

and a current step arises in the circuit, but the potential difference across the capacitor at this instant is $V_c \sim \varphi = 0$. This gives us the constant of integration in Eq. (1.7), so the particular solution of Eq. (1.6) is

$$\varphi = \mu\exp(\mu^2)\sqrt{\pi}\,[\mathrm{erf}(\mu_0) - \mathrm{erf}(\mu)]. \tag{1.9}$$

It is clear from Eqs. (1.5) and (1.8) that μ_0 and μ are related by

$$\mu = \mu_0[1 - (1-z)X] \quad (X = Dtl_0^{-1}) \tag{1.10}$$

in which X is a dimensionless coordinate ranging from 0 to 1 as t varies, μ running from μ_0 to $\mu_0 z$.

Finally, from Eq. (1.3) we have

$$I = 1 - \mu\exp(\mu^2)\sqrt{\pi}\,[\mathrm{erf}(\mu_0) - \mathrm{erf}(\mu)]. \tag{1.11}$$

Figure 4 shows $I = f(X)$ as solid lines 1–5 for μ_0 of 0.5, 1, 2, 4, and 10 respectively.

Case 2. The material behind the front is a good conductor, i.e., $R_{c2} \gg R_2$ and $R = R_i + R_e \gg R_2$.

Here again the processes in Fig. 2 are described by the circuit of Fig. 3b, and the equations remain the same, except that the capacitance C is the C_1 of Fig. 3a. This alters the form of

$$\mu_0 = 2l_0\sqrt{\pi}\,(2RD\varepsilon_1)^{-1/2} \tag{1.12}$$

and Eq. (1.10) becomes

$$\mu = \mu_0(1 - X) \qquad (X = Dtl_0^{-1}). \tag{1.13}$$

The range in μ is from μ_0 to 0 as X goes from 0 to 1; Fig. 4 (broken lines) shows the results from Eq. (1.11) for this case.

In both cases the value Z = 0.5 has been used.

2. ANALYSIS OF RESULTS

1) The shape of the $I = f(X)$ curves in Fig. 2 is readily explained, since at t = 0 the following operations are performed simultaneously in the circuit of Fig. 3b: the source of emf is switched on and the effective resistance of C drops from $R_c = \infty$ to some finite value R_{c0}. The emf produces a current step $i = E_0/R$ (I = 1), with subsequent decay governed by the relaxation time. Further, the change in R_c produces a current step $i = E_0(R + R_c)^{-1}$ or in dimensionless terms,

$$I = (1 + 2\mu^2)^{-1}$$
$$(I = (2\mu^2)^{-1} \text{ for } 2\mu^2 \gg 1). \tag{2.1}$$

The shape of the I(X) curves is determined by the joint action of these two factors.

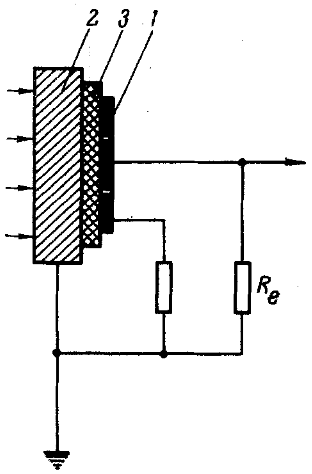

Fig. 1

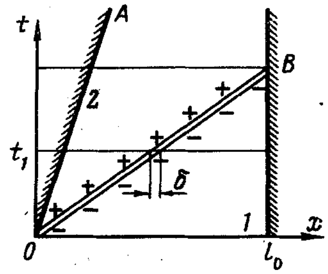

Fig. 2

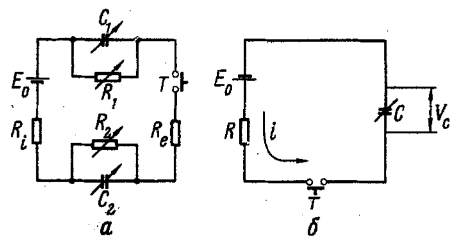

Fig. 3

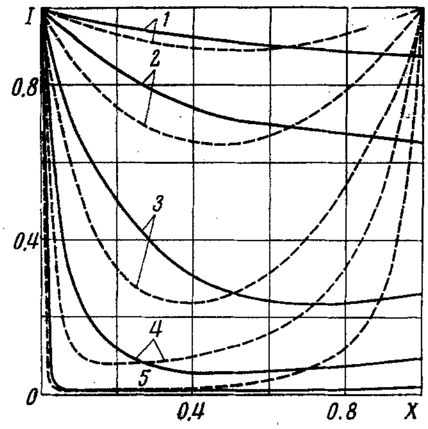

Fig. 4

If RC_0 is small relative to l_0/D (μ_0 large), the first factor may be neglected to a first approximation. Then the initial condition for Eq. (1.7) will be $\varphi = 1$ at $t = 0$, so

$$I = 1 - \mu \exp(\mu^2)\{\mu_0^{-1} \exp(-\mu_0^2) + \sqrt{\pi}\,[\operatorname{erf}(\mu_0) - \operatorname{erf}(\mu)]\}\ . \quad (2.2)$$

We stop the series expansion of $\operatorname{erf}(\mu)$ at terms of the second order in μ to get

$$I = (2\mu^2)^{-1}$$

which agrees with (2.1).

2) We put the ratio of R_{C2} and R_2 from Eq. (1.1) as $4\pi t/\rho_2\varepsilon_2$.

Case 1 corresponds to $4\pi t/\rho_2\varepsilon_2 \ll 1$, while case 2 corresponds to $4\pi t/(\rho_2\varepsilon_2) \gg 1$. We assume for case 1 that $t = l_0/D \approx 1$ μsec, while $\varepsilon_2/4\pi$ is 0.5; then $\rho_2 \gg 2 \times 10^6$ ohm · cm. The condition $4\pi t/\rho_2\varepsilon_2 \gg 1$ for the second case can be obeyed only for $t > t_0$, in which t_0 is some minimum time, which we may reasonably take as the resolving time of the apparatus (about 10^{-8} sec), and so $\rho_2 \ll 2 \times 10^4$ ohm · cm is not covered by solutions 1 and 2.

3) Consider the region of μ_0 large for case 1. The circuit current is given by Eq. (2.1). By specifying the ratio of the currents for two given values of X (or t) on each of the I(X) curves of Fig. 4, we get an expression for z:

$$z = \frac{1}{4K - 1}(1 \pm 2\sqrt{K}) \quad (2.3)$$

in which K is the ratio of the current at $X = 1$ to that at $X = 0.5$; $K > 1$ for this range μ_0 (Fig. 4), while z is positive, so Eq. (2.3) may be put as

$$z = (2\sqrt{K} - 1)^{-1}\ . \quad (2.4)$$

4) The inverse relation of the initial current step to l_0 [1] becomes an inverse-square relation for μ_0 large as (2.1) shows.

5) The second formula of (2.1), put in the form $i = E_0 dC/dt$, is the basic formula for the capacity-transducer method [2].

This feature may be of value in determining ε_2 for dielectrics that do not produce an adequate E_0, e.g., nonpolar dielectrics or polar ones subject to only small σ (for instance, the lower limit of detection for lucite is given [1] as a pressure of 40 kbar).

Naturally, an external source of emf is needed in these cases.

6) The solution for the circuit of Fig. 3a can, in principle, give information on ρ_2, the specific bulk resistance behind the shock wave.

Then ε_2 is readily found from Eq. (1.5). In case 2 the parameters to be determined are E_0 and R. Figure 4 shows that K is $(I_{X=0.5})^{-1}$, which Eq. (1.11) shows to be uniquely related to μ_0, and the latter defines R via Eq. (1.12). There is no difficulty in finding E_0.

7) This solution for substances with orientation polarization is correct also for substances with ionic or atomic polarization, since no assumption is made about the nature of the effect responsible for the emf.

REFERENCES

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